Pergamon

Int. J. Heat Mass Transfer. Vol. 40, No. 14, pp. 3375–3385, 1997

Printed in Great Britain
0017–9310/97 $17.00+0.00

PII: S0017–9310(96)00384–5

Modelling of steady-state two-phase bubbly flow through a sudden enlargement

A. ATTOU, M. GIOT and J. M. SEYNHAEVE

Unité Thermodynamique et Turbomachines, Université catholique de Louvain, Place du Levant 2, 1348 Louvain-la-Neuve, Belgium

(*Received* 10 *November* 1995 *and in final form* 28 *October* 1996)

Abstract—A global formulation of the conservation laws (mass, momentum and energy) is applied to a two-phase, two-component flow through a sudden enlargement. The assumption of thermal equilibrium of the phases is acceptable. However, due to the difference in the mechanical inertia of the phases, the kinematic non-equilibrium effect has to be taken into account. In order to determine the role of mechanical non-equilibrium, two assumptions are made, which can be considered as two limited cases. Firstly, an infinite momentum transfer coefficient (mechanical equilibrium model) is assumed: an analytical solution can be obtained when the gas is ideal. Secondly, no momentum transfer can occur between phases (mechanical frozen model): an approximate analytical solution is obtained in this case. The comparison in terms of singular pressure variations between the results of these two models and the experimental data of other authors for air–water mixtures shows clearly that both models indeed simulate two extreme conditions. New experimental data were obtained for two-phase air–water bubbly flow through an axisymmetric and horizontal sudden enlargement. A new physical model approximately taking into account the effects of the interfacial drag of the bubbles is developed, and compared favourably with the data in the literature and the new data. This model shows a rather limited dependence with respect to the reduced bubble diameter. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

Different approaches can be considered to describe two-phase flow through a sudden enlargement. Such singularity is often encountered in the safety systems of process engineering, such as a pressure relief system. Generally, these circuits have the shape of a wire, which means that they have a characteristic length dominating with respect to the other dimensions. In this case, a global approach is the most suitable way to study the expansion as an element of a circuit, when a calculation has to be made on the scale of the whole circuit. Moreover, it enables one to easily adapt it to a computer code of a 1-D flow.

The behaviour of two-phase flow through a sudden enlargement has been the subject of several experimental and theoretical investigations. An important parameter which characterizes this type of singularity is the global singular pressure variation. Several analytical methods of calculating this quantity exist in the literature [1–4]. Some of these methods use an empirical correction, while others are incomplete and need closing relationships for the global void fraction. This could explain the limited character of the domain of application of each correlation. The incoherences observed by some authors [3, 5] can be also explained by the fact that these correlations do not take into account the structure of the flow. These previous simple correlations are often inaccurate.

The aim of this paper is to identify the influence of the kinematic non-equilibrium of the phases by solving some simplified models based on limited assumptions. Analytical solutions of these simplified models can be obtained if the gas is assumed to be ideal. A bubbly flow model is then established in order to approximately take into account the interfacial drag. The system of non-linear equations which are deduced needs an iterative numerical method for its solution. This model, which does not seem to be very sensitive to the diameter of bubbles over a wide physical interval, is compared with the two simplified models. The bubbly model is finally valided by means of the experimental data of the literature, and new data that were obtained for bubbly air–water flows through axisymmetric and horizontal sudden enlargement.

2. BASIS FORMULATION

Consider a gas–liquid two-component subsonic flow through a sudden enlargement (Fig. 1). Isolate a volume V of fluid limited by the closed surface A made of:

—the upstream section A_1 and the downstream section A_2 selected in such a way that the flow is fully developed;
—the lateral surface of the wall, A_p;
—the surface of the cross section, A_0.

Apply the global balance laws to the part of the control volume V occupied on average by phase K. The mass balance of phase K can be written as

NOMENCLATURE

- A_1, A_2 geometrical sections defined in Fig. 1 [m^2]
- A_0, A_p material surfaces defined in Fig. 1 [m^2]
- C empirical parameter in the correlation of Chisholm *et al.*
- C_D bubble drag coefficient
- C_{DB} drag coefficient of an isolated bubble
- C_{pK} specific heat of phase K [J kg^{-1} K^{-1}]
- $C_{p,m}$ average quantity, defined by $xC_{pG}+(1-x)C_{pL}$ [J kg^{-1} K^{-1}]
- C_T coefficient defined by $\Delta p_{sing}/(G_1^2/2\rho_H)$
- E_K specific total energy of phase K [J kg^{-1}]
- f factor in equation (22)
- F_1, F_2 momentum flux [equation (15)] [Pa]
- g gravitational acceleration [m s^{-2}]
- G total mass velocity defined by M/A [kg m^{-2} s^{-1}]
- h_K specific enthalpy of phase K [J kg^{-1}]
- H step height, defined by $(D_2-D_1)/2$, where D is the pipe diameter [m]
- K empirical parameter in the correlation of Wadle
- l_2, l_0 dimensionless lengths, defined by L_2/H and L_0/H, respectively
- L_0 length defined by $\int_0^{L_0} \tau_D \, dz = 0$ [m]
- L_2 length from the enlargement to obtain the fully established flow [m]
- M total mass flow rate [kg s^{-1}]
- $\mathbf{n}$ unit vector normal to the closed surface A, oriented outwards
- $\mathbf{q}_K$ heat flux density exchanged with phase K [W m^{-2}]
- R ideal gas constant [J kg^{-1} K^{-1}]
- $\mathbf{V}_i$ interface velocity [m s^{-1}]
- $\mathbf{V}_K$ velocity of phase K [m s^{-1}]
- V_{SL} superficial velocity of liquid, defined by Q_L/A, where Q_K is the volumetric flow rate of phase K [m s^{-1}]
- R_B bubble radius [m]
- Re_B bubble Reynolds number, defined by $2R_B\rho_L(V_G-V_L)/\mu_L$, where μ_L is the liquid dynamic viscosity
- Re_L Reynolds number of the liquid flow, defined by $(1-x)GD/\mu_L$
- T absolute temperature [K]
- $\bar{\bar{T}}_K$ total stress tensor of phase K: $\bar{\bar{T}}_K = \bar{\bar{\tau}}_K - p_K\bar{\bar{I}}$, where $\bar{\bar{\tau}}_K$ is the viscous stress tensor, and p_K is the pressure of phase K [Pa]
- S velocity ratio defined by V_G/V_L
- x quality
- X_K phase density function (equal to 1 or 0, the local point being occupied or not by phase K)
- X_t parameter defined by $[(1-x)/x]\sqrt{(\rho_G/\rho_L)}$.

Greek symbols

- α global void fraction
- α_h volumetric quality, defined by $Q_G/(Q_G+Q_L)$
- Δp_{sing} singular pressure variation calculated by equation (28) [Pa]
- θ angle between $\mathbf{1}_z$ and $\mathbf{g}$ (Fig. 1)
- λ_L single-phase friction factor (liquid flow)
- ξ surface tension [Pa m^{-1}]
- ρ_K density of phase K [kg m^{-3}]
- ρ_H homogeneous mixture density, defined by $1/(x/\rho_G+(1-x)/\rho_L)$ [kg m^{-3}]
- ρ_m average mixture density, defined by $\alpha\rho_G+(1-\alpha)\rho_L$ [kg m^{-3}]
- σ area ratio of the enlargement, defined by A_1/A_2
- τ_{fr} axial projection of the resulting forces on the lateral wall A_p [N]
- ϕ_L^2 frictional pressure loss multiplier
- ϕ_{Lo}^2 multiplier factor of singular pressure variation
- χ^2 Lockhart–Martinelli parameter
- Ω parameter of the MEM solution (Section 3.1) [Pa^2].

$$\frac{d}{dt}\int_V \overline{X_K\rho_K}\, dV + \int_A \overline{X_K\rho_K \mathbf{V}_K \mathbf{n}}\, dA + \overline{\int_{A_{int}} \rho_K(\mathbf{V}_K-\mathbf{V}_i)\mathbf{n}_K\, dA} = 0 \quad (1)$$

where X_K is the characteristic presence function of phase K (0 or 1), $\mathbf{V}_K$ is the velocity vector of phase K, ρ_K is the density of phase K, $\mathbf{n}$ is the unit vector normal to A, oriented towards the outside of volume V, $\mathbf{n}_K$ is the unit vector normal to the interface A_{int}, oriented towards the outside of the phase K, and $\mathbf{V}_i$ is the interface velocity vector.

The momentum balance equation for phase K leads to

$$\frac{d}{dt}\int_V \overline{X_K\rho_K\mathbf{V}_K}\, dV + \int_A \overline{X_K[\rho_K(\mathbf{V}_K\mathbf{n})\mathbf{V}_K - \bar{\bar{T}}_K\mathbf{n}]}\, dA + \overline{\int_{A_{int}} [\rho_K(\mathbf{V}_K\mathbf{n}_K)(\mathbf{V}_K-\mathbf{V}_i) - \bar{\bar{T}}_K\mathbf{n}_K]\, dA} = \int_V \overline{X_K\rho_K\mathbf{g}}\, dV \quad (2)$$

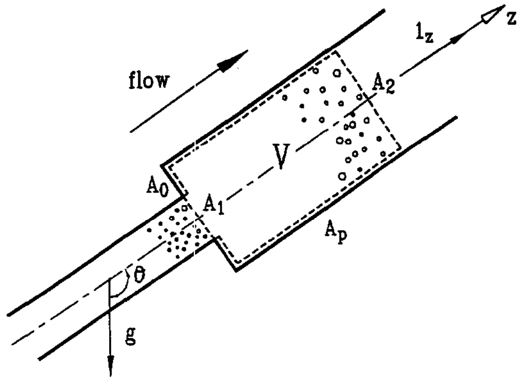

Fig. 1. Schematic representation of a sudden enlargement.

where $\bar{T}_K$ is the stress tensor of phase K, and g is the gravitational acceleration.

The energy balance of phase K can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \overline{X_K\rho_K E_K}\,\mathrm{d}V + \int_A \overline{X_K\rho_K(\mathbf{V}_K\mathbf{n})E_K}\,\mathrm{d}A + \int_{A_{\mathrm{int}}} \overline{[\rho_K(\mathbf{V}_K-\mathbf{V}_\mathrm{i})\mathbf{n}_K E_K - (\bar{T}_K\mathbf{n}_K)\mathbf{V}_K + \mathbf{q}_K\mathbf{n}_K]}\,\mathrm{d}A = \int_V \overline{X_K\rho_K \mathbf{g}\mathbf{V}_K}\,\mathrm{d}V + \int_A \overline{X_K(\bar{T}_K\mathbf{V}_K)\mathbf{n}}\,\mathrm{d}A - \int_A \overline{X_K\mathbf{q}_K\mathbf{n}}\,\mathrm{d}A \quad (3)$$

where E_K is the specific energy of phase K equal to $u_K + (V_K^2/2)$ (u_K is the internal energy per unit mass of phase K), and $\mathbf{q}_K$ is the heat flux density exchanged with phase K. A bar above a quantity means that this quantity is averaged over some time interval large enough to smooth out the instantaneous fluctuations. Hereafter, a quasi-steady flow is considered, which is often the case in practice, i.e. a flow for which

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \overline{X_K f_K}\,\mathrm{d}V = 0$$

where f_K is a conservative quantity (ρ_K, $\rho_K\mathbf{V}_K$ or $\rho_K E_K$). Moreover, it is assumed that the flow is adiabatic.

The mass quality remains unchanged for a two-component system : equation (1) for the mass balance of phase K becomes

$$\int_{A_1} \overline{X_K\rho_K(\mathbf{V}_K\mathbf{I}_z)}\,\mathrm{d}A = \int_{A_2} \overline{X_K\rho_K(\mathbf{V}_K\mathbf{I}_z)}\,\mathrm{d}A \quad (K = G \text{ or } L). \quad (4)$$

By adding up equations (2) for each phase, and taking into account the local interfacial condition [6]

$$\sum_K [\rho_K(\mathbf{V}_K\mathbf{n}_K)(\mathbf{V}_K-\mathbf{V}_\mathrm{i}) - \bar{T}_K\mathbf{n}_K] = 0$$

the momentum balance of the mixture becomes

$$\sum_K\int_{A_2} \overline{X_K\rho_K(\mathbf{V}_K\mathbf{I}_z)(\mathbf{V}_K\mathbf{I}_z)}\,\mathrm{d}A - \sum_K\int_{A_1} \overline{X_K\rho_K(\mathbf{V}_K\mathbf{I}_z)(\mathbf{V}_K\mathbf{I}_z)}\,\mathrm{d}A - \sum_K\int_A \overline{X_K(\bar{T}_K\mathbf{n})\mathbf{I}_z}\,\mathrm{d}A = \sum_K\int_V \overline{X_K\rho_K(\mathbf{g}\mathbf{I}_z)}\,\mathrm{d}V. \quad (5)$$

By adding up equations (3) for each phase, and taking into account the local interface condition [6]

$$\sum_K [\rho_K(\mathbf{V}_K-\mathbf{V}_\mathrm{i})\mathbf{n}_K E_K - (\bar{T}_K\mathbf{n}_K)\mathbf{V}_K + q_K\mathbf{n}_K] = 0$$

the energy balance of the mixture becomes

$$\sum_K\int_{A_2} \overline{X_K\rho_K(\mathbf{V}_K\mathbf{I}_z)E_K}\,\mathrm{d}A - \sum_K\int_{A_1} \overline{X_K\rho_K(\mathbf{V}_K\mathbf{I}_z)E_K}\,\mathrm{d}A = \sum_K\int_V \overline{X_K\rho_K\mathbf{g}\mathbf{V}_K}\,\mathrm{d}V + \int_A \overline{X_K(\bar{T}_K\mathbf{V}_K)\mathbf{n}}\,\mathrm{d}A. \quad (6)$$

Equations (4)–(6) are developed in an algebraic form and are submitted to the following simplifying assumptions:

H1—the turbulence terms are neglected [7],
H2—the spatial distribution parameters are assumed to be equal to 1,
H3—the effects linked to the superficial tension are neglected (consequently, one can assume the equilibrium of the averaged phasic pressures: $p_G = p_L = p$),
H4—the pressure is approximately uniform in the section of the singularity,
H5—the thermal equilibrium between the phases can be considered as acceptable to a first approximation ($T_\mathrm{G} = T_\mathrm{L} = T$),
H6—the gaseous phase is considered as ideal gas ($\rho_\mathrm{G} = p/RT$, where R is the ideal gas constant), the liquid is incompressible (ρ_L = constant), and the specific heat $C_{\mathrm{p}K}$ of each phase is constant. The enthalpy variation of the gas is given by

$$h_{\mathrm{G2}} - h_{\mathrm{G1}} = C_{\mathrm{pG}}(T_2 - T_1)$$

and the enthalpy variation of the liquid by

$$h_{\mathrm{L2}} - h_{\mathrm{L1}} = \frac{p_2 - p_1}{\rho_\mathrm{L}} + C_{\mathrm{pL}}(T_2 - T_1).$$

Taking into account assumptions H1–H4, equations (4)–(6) are reduced to the following set:

$$\sigma\alpha_1\rho_{G1}V_{G1} = \alpha_2\rho_{G2}V_{G2}$$
$$\sigma(1-\alpha_1)\rho_L V_{L1} = (1-\alpha_2)\rho_L V_{L2}$$
$$p_1+\sigma[\alpha_1\rho_{G1}V_{G1}^2+(1-\alpha_1)\rho_L V_{L1}^2]+\int_{(1)}^{(2)}\rho_m g_z\,dz - \frac{\tau_{fr}}{A_2} = p_2+[\alpha_2\rho_{G2}V_{G2}^2+(1-\alpha_2)\rho_L V_{L2}^2]$$
$$\sigma[\alpha_1\rho_{G1}(h_{G1}+\tfrac{1}{2}V_{G1}^2)V_{G1}+(1-\alpha_1)\rho_L(h_{L1}+\tfrac{1}{2}V_{L1}^2)V_{L1}] + \sigma G_1 L_2 g_z = \alpha_2\rho_{G2}(h_{G2}+\tfrac{1}{2}V_{G2}^2)V_{G2} + (1-\alpha_2)\rho_L(h_{L2}+\tfrac{1}{2}V_{L2}^2)V_{L2} \quad (7)$$

where $\sigma = A_1/A_2$ denotes the area ratio of the enlargement, $\rho_m = \alpha\rho_G+(1-\alpha)\rho_L$ is the averaged density of the mixture, L_2 is the distance between the cross sections A_1 and A_2, τ_{fr} is the projection in the z-direction of the resulting wall forces on the lateral wall A_p, and $G_1 = M/A_1$ is the mass velocity at the inlet, where M is the total mass flow rate. The sign for τ_{fr} has been selected such that $\tau_{fr} > 0$.

3. SIMPLIFIED MODELS

The difference between the mechanical inertia of both phases leads to the consideration of a kinematic non-equilibrium. The slip between phases is mainly due to the non-perfect momentum transfer at the interface. This momentum transfer is influenced by several physical parameters, such as the topology of the interface, the physical properties of the phases, the mass flux, and the quality. In order to estimate the role of this type of non-equilibrium on the global characteristics of the flow through an expansion, two models will be analysed: the mechanical equilibrium model (MEM) (no slip between phases), and the mechanical frozen model (MFM) (no momentum transfer between phases).

3.1. *MEM*

One assumes that the momentum transfer coefficient between the phases in the control volume is infinite. This is equivalent to the assumption that no slip exists between phases. Thus

$$V_{Gi} = V_{Li} = V_i \quad (i = 1,2). \quad (8)$$

By eliminating the void fraction between equations (7) and (8), the set of equations is reduced to a simpler one, where the variables are p, T and V. If this set is made explicit with the quality x, which is constant, and taking into account assumptions H5 and H6, one finds that

$$\frac{V_2}{\dfrac{xRT_2}{p_2}+\dfrac{1-x}{\rho_L}} = \sigma G_1$$
$$p_2+\sigma G_1 V_2 = P_1$$
$$\frac{1}{2}V_2^2+C_{p,m}T_2+\frac{1-x}{\rho_L}p_2 = E_1 \quad (9)$$

where

$$G_1 = \frac{V_1}{\dfrac{x}{\rho_{G1}}+\dfrac{1-x}{\rho_L}}$$

is the mass velocity of the mixture at A_1,

$$P_1 = p_1+\sigma G_1 V_1$$

is the total pressure at A_1,

$$E_1 = \frac{1}{2}V_1^2+C_{p,m}T_1+\frac{1-x}{\rho_L}p_1$$

is the total energy per unit mass at A_1, and

$$C_{p,m} = xC_{pG}+(1-x)C_{pL}.$$

In this simplified approach, the wall friction term at the wall A_p and the gravity term are neglected. Although the set of equations (9) is non-linear, it accepts an exact analytical solution given by

$$p_2 = \frac{\left(\dfrac{C_{p,m}}{Rx}-1\right)\left[P_1-\left(\dfrac{1-x}{\rho_L}\right)(\sigma G_1)^2\right]+\Omega^{1/2}}{\dfrac{2C_{p,m}}{Rx}-1}$$
$$V_2 = \frac{P_1-p_2}{\sigma G_1}$$
$$T_2 = \left[\frac{P_1-p_2}{(\sigma G_1)^2}-\frac{1-x}{\rho_L}\right]\frac{p_2}{Rx}$$
$$\alpha_2 = \frac{1}{1+\left(\dfrac{1-x}{x}\right)\dfrac{p_2}{RT_2\rho_L}} \quad (10)$$

where

$$\Omega \equiv \left(1-\frac{C_{p,m}}{Rx}\right)^2\left[\left(\frac{1-x}{\rho_L}\right)(\sigma G_1)^2-P_1\right]^2 - \left(1-2\frac{C_{p,m}}{Rx}\right)[P_1^2-2(\sigma G_1)^2E_1].$$

The mixture is equivalent to a single-phase fluid whose properties are the averages of the corresponding properties of each phase. In the MEM, the upstream velocity is estimated by

$$V_1 = \frac{G_1}{\rho_{H,1}}$$

where

$$\rho_{H,1} = \left(\frac{x}{\rho_{G1}}+\frac{1-x}{\rho_L}\right)^{-1}$$

is the homogeneous density.

3.2. *MFM*

The momentum transfer between the phases is neglected. This means that the phases are completely disconnected, and, thus, each phase flows according to its own mechanical inertia. By expressing the momentum balance equation of the gas phase in which the interfacial transfer term vanishes, it is found that

$$\int_{A_2} \overline{X_G \rho_G V_G^2}\,\mathrm{d}A - \int_{A_1} \overline{X_G \rho_G V_G^2}\,\mathrm{d}A - \int_A \overline{X_G (\bar{\bar{T}}_G \mathbf{n}) \mathbf{I}_z}\,\mathrm{d}A = \int_V \overline{X_G \rho_G g_z}\,\mathrm{d}V. \quad (11)$$

Taking into account assumptions H1–H4, the balance equation (11) is reduced to

$$\sigma\left\{\left[\alpha_1 + \left(\frac{1-\sigma}{\sigma}\right)\alpha_0\right]p_1 + \alpha_1 \rho_{G1} V_{G1}^2\right\} = \alpha_2 (p_2 + \rho_{G2} V_{G2}^2). \quad (12)$$

No interaction between the phases corresponds to the condition where the momentum flux defined by

$$\alpha \rho_G V_G^2 + (1-\alpha)\rho_L V_L^2$$

is minimum. In this case, the velocity ratio is given by

$$\frac{V_{G2}}{V_{L2}} = \left(\frac{\rho_L}{\rho_{G2}}\right)^{1/2}. \quad (13)$$

The void fraction in the separated zone must be considered as a variable of the MFM. By taking into account assumptions H5 and H6, an approximate solution of the system of equations (7), (12) and (13) can be obtained if the superficial velocity of the liquid is not too high (subsonic flow). One can then assume that

$$\frac{V_{G2}}{V_{L2}} \cong \left(\frac{\rho_L}{\rho_{G1}}\right)^{1/2} \equiv S. \quad (14)$$

The approximate variable vector is given by

$$V_{L2} \cong \frac{-B + (B^2 - 4AC)^{1/2}}{2A}$$

$$V_{G2} \cong S V_{L2}$$

$$p_2 \cong (p_1 + \sigma F_1) - F_2$$

$$T_2 \cong T_1 \frac{p_2}{p_1} \frac{V_{G2}}{V_{G1}} \frac{1}{\sigma}$$

$$\alpha_2 \cong \frac{1}{1 + \left(\frac{1-x}{x}\right)\frac{p_2}{RT_2 \rho_L} S}$$

$$\alpha_0 \cong \left\{\left[\alpha_2 p_2 \left(1 + \frac{V_{G2}^2}{RT_2}\right)\frac{1}{\sigma} - \alpha_1 \rho_{G1} V_{G1}^2\right]\frac{1}{p_1} - \alpha_1\right\}\left(\frac{\sigma}{1-\sigma}\right) \quad (15)$$

where

$$F_1 \equiv \alpha_1 \rho_{G1} V_{G1}^2 + (1-\alpha_1)\rho_L V_{L1}^2$$

$$F_2 \equiv \sigma G_1 [x V_{G2} + (1-x) V_{L2}]$$

$$A = \frac{1}{2}(xS^2 + 1 - x) - \frac{C_{p,m} T_1 S G_1 (xS + 1 - x)}{p_1 V_{G1}}$$

$$B = \frac{S C_{p,m} T_1}{V_{G1}}\left(\frac{F_1}{p_1} + \frac{1}{\sigma}\right) - \left(\frac{1-x}{\rho_L}\right)\sigma G_1 (xS + 1 - x)$$

$$C = \left(\frac{1-x}{\rho_L}\right)\sigma F_1 - C_{p,m} T_1.$$

Solution (15) is verified by comparing it with the numerical solution of the set of equations (7), (12) and (13) for an air–water flow at ambient pressure and temperature, qualities ranging between 10^{-3} and 10^{-2} ($\alpha_1 \sim 0.2$–0.4), and with different area ratios. Both solutions agree within an accuracy ranging from 0.1 to 5% for a not too high V_{SL}. Thus, result (15) can be used as a solution of the MFM. For this model, each phase behaves independently of the other. The velocity non-equilibrium, mainly fixed by the pressure gradient of the flow, becomes maximum. The two-phase flow is equivalent to two independent single-phase streams. The velocity of each phase is estimated by the following relationships:

$$V_{G1} = \frac{x G_1}{\alpha_1 \rho_{G1}} \quad \text{and} \quad V_{L1} = \frac{(1-x) G_1}{(1-\alpha_1)\rho_L}.$$

In the MFM, the global void fraction α_1 is evaluated by the velocity ratio

$$\frac{V_{G1}}{V_{L1}} = \left(\frac{\rho_L}{\rho_{G1}}\right)^{1/2}$$

which corresponds to the minimum momentum flux.

Neglecting the wall friction term and the gravity term, a multiplier factor of singular pressure variation is defined by

$$\phi_{Lo}^2 = \frac{p_2 - p_1}{\Delta p_{Lo}} \quad (16)$$

where

$$\Delta p_{Lo} = \sigma(1-\sigma)\frac{G_1^2}{\rho_L} \quad (17)$$

is the singular pressure variation in the flow of liquid only.

Pressure p_2 which appears in equation (16) is directly deduced from solution (10) for the MEM, or from solution (15) for the MFM. The results of the comparison are presented later in this paper.

4. BUBBLY FLOW MODEL

The variables describing the flow have been selected as follows: p, T, V_G, V_L and α. Thus, five equations are needed to solve the problem. The four available equations are given by set (7).

A complementary equation will be given by expressing the momentum balance equation of a given phase, for example the gaseous phase. It is necessary to introduce a particular structure of the flow in order to approximately take into account the interaction forces between the phases. Consider a bubbly flow; the bubbles will be assumed to be spherical, and with a small diameter with respect to the pipe diameter in order to neglect the effects of fragmentation and coalescence. By expressing the momentum balance equation integrated over the volume of a bubble, Berne [8] obtains the following momentum equation, in the steady-state condition:

$$\rho_G V_G \frac{dV_G}{dz} = -\left[\frac{dp}{dz} + \frac{1}{2}\rho_L V_G \frac{d}{dz}(V_G - V_L) + \frac{3}{2}\rho_L (V_G - V_L)\frac{dV_L}{dz} + \rho_G g_z + \frac{3}{2R_B}\rho_L (V_G - V_L) V_G \frac{dR_B}{dz} + \frac{3}{8R_B} C_D \rho_L (V_G - V_L)|V_G - V_L|\right] \quad (18)$$

where R_B is the bubble radius, and C_D is the drag coefficient of the bubble.

Although this equation is established for an isolated bubble, it remains suitable for a flow with many bubbles as long as there is a low void fraction, and the bubble radius remains small with respect to the pipe diameter.

In this condition, the bubble interaction effects can be neglected as a first approximation. Nevertheless, for higher void fractions, these effects have to be taken into account. The momentum equation (18) is applied at cross section A_2, where the flow is fully established. Due to the weak mechanical inertia of the gaseous phase, the term $\rho_G V_G \, dV_G/dz$ is small with respect to the pressure gradient. Due to the incompressibility of the liquid phase, the acceleration term of the liquid remains also small with respect to the pressure gradient. The term of the bubble growth is mainly due to the phase change; this term can be neglected for a two-component system. Thus, the following equation:

$$-\left(\frac{dp}{dz}\right)_2 = \frac{3}{8R_{B2}} C_{D2}\rho_L (V_{G2} - V_{L2})|V_{G2} - V_{L2}| + \frac{1}{2}\rho_L V_{G2}\frac{d}{dz}|(V_G - V_L)|_2 - \rho_G g_z \quad (19)$$

is obtained. The momentum law of the mixture expressed in the downstream section A_2 can be written as

$$-\left(\frac{dp}{dz}\right)_2 = \left|\left(\frac{dp}{dz}\right)_{f,2}\right| + \alpha_2 \rho_{G2} V_{G2}\left(\frac{dV_G}{dz}\right)_2 + (1-\alpha_2)\rho_L V_{L2}\left(\frac{dV_L}{dz}\right)_2 - \rho_m g_z \quad (20)$$

where $(dp/dz)_f$ is the two-phase pressure gradient due to wall friction.

Due to the weak mechanical inertia of the gaseous phase, and due to the incompressibility of the liquid,

$$-\left(\frac{dp}{dz}\right)_2 \cong \phi_{L2}^2(\chi^2)\cdot\left|\left(\frac{dp}{dz}\right)_{L,2}\right| - \rho_m g_z \quad (21)$$

is given approximately, where

$$\phi_L^2 = \frac{(dp/dz)_f}{(dp/dz)_L}$$

is the multiplier factor for the wall friction,

$$\chi^2 = \frac{(dp/dz)_L}{(dp/dz)_G}$$

is the Lockhart–Martinelli parameter, where $(dp/dz)_K$ is the pressure gradient due to the wall friction for the flow of phase K only.

Berne [8] has observed that the acceleration term related to the phases for a steam–water flow remains at a maximum of 10% of the pressure gradient. This minor contribution is mainly due to the bubble growth induced by the vaporization during the expansion. This bubble growth is significant, and leads to large differences in phase accelerations due to the large mechanical inertia difference. In the case of a two-component system, the interface mass transfer is zero, and one can assume the relative acceleration term is less than 10% of the pressure gradient. To check this, an isoquality flow at ambient pressure and temperature ($p \sim 2\times10^5$ Pa and $T \sim 293$ K) is considered. The mass quality is selected as 10^{-3}, the void fraction is 0.2, and the mass velocity changes between 10^3 and 5×10^3 kg m^{-2} s^{-1}. Some simple calculations have shown that, in the most unfavourable case, which is the case where the relative variation of the void fraction is most important, the relative acceleration term is limited to about 2–3% of the pressure gradient (dp/dz is evaluated by the usual friction factor formula). The contribution of the relative acceleration will thus be neglected, and equation (19) becomes purely algebraic. Further, the mutual interaction of the bubbles tends to decrease the relative velocity between the bubble and the liquid with respect to the case where the bubble would be isolated in the liquid. This collective effect tends to increase the bubble drag, and can be approximately taken into account to enlarge the application domain of the momentum equation for larger void fractions:

$$C_D = C_{DB} f(\alpha) \quad (22)$$

where C_{DB} denotes the drag coefficient of an isolated bubble, and $f(\alpha)$ is the corrective factor function of the void fraction: the Wen–Yu correction is used (recommended by Wallis [9]), which is

$$f(\alpha) = \frac{1}{(1-\alpha)^{4.7}}.$$

Two types of corrections have been used to take into account perturbation due to the set of bubbles: first of all, $(dp/dz)_f$ is more important in a two-phase regime than in a single-phase liquid flow. The difference is more important with a higher void fraction, and is taken into account by means of the multiplier factor Φ_L^2. Then, the drag coefficient is also influenced by the bubble interaction, which increases with the void fraction. The difference between the drag coefficient of an isolated bubble and the drag coefficient in the bubbly flow is introduced by means of a corrective factor, $f(\alpha)$.

The wall friction force τ_{fr} which appears in set (7) can be evaluated as follows:

$$\tau_{fr} = \int_{A_p} \tau_D \, d\Sigma = 2\pi R_2 (L_2 - L_0)\tau_{D2} \qquad (23)$$

where L_0 denotes the distance from the enlargement such that the resultant force at the wall due to the friction is zero, which means that $\int_0^{L_0} \tau_D \, dz = 0$; R_2 is the radius of the downstream duct.

From equation (23):

$$\frac{\tau_{fr}}{A_2} = \frac{2}{R_2}(L_2 - L_0)\tau_{D2}$$

is deduced, or, in another form,

$$\frac{\tau_{fr}}{A_2} = (1-\sqrt{\sigma})\sigma^2 G_1^2 \frac{(1-x)^2}{\rho_L}(l_2 - l_0)\lambda_{L2}(Re_L)\phi_{L2}^2(\chi^2) \qquad (24)$$

where $l_2 = L_2/H$ and $l_0 = L_0/H$, where $H = (R_2 - R_1)$, λ_L is the single-phase friction factor, and $Re_L = [2(1-x)GR]/\mu_L$ is the Reynolds number of the liquid only.

The integral gravity term, which appears in set (7), can be evaluated by assuming that the density of mixture averaged through the cross section of the downstream pipe is not disturbed too much. Then:

$$\int_{(1)}^{(2)} \rho_m g_z \, dz \cong \frac{1}{2}(\rho_{m1}^+ + \rho_{m2}) g_z L_2 \qquad (25)$$

where

$$\rho_{m1}^+ = \sigma\alpha_1\rho_{G1} + (1-\sigma\alpha_1)\rho_L \quad \text{and}$$

$$\rho_{m2} = \alpha_2\rho_{G2} + (1-\alpha_2)\rho_L.$$

The average density through the cross section located just downstream from the enlargement, ρ_{m1}^+, is evaluated according to several experimental observations [5, 7]. In fact, these measurements have shown that there are only a few bubbles in the step section $A_0(\alpha_0 \ll \alpha_1)$.

Taking into account equations (19), (21), (22), (24), (25) and set (7), the five-equation model of the bubbly flow regime through the enlargement can be written as follows:

$$\alpha_2\rho_{G2}V_{G2} = \sigma\alpha_1\rho_{G1}V_{G1}$$

$$(1-\alpha_2)\rho_L V_{L2} = \sigma(1-\alpha_1)\rho_L V_{L1}$$

$$\begin{aligned}
&(p_2 - p_1) + (\alpha_2\rho_{G2}V_{G2}^2 + (1-\alpha_2)\rho_L V_{L2}^2) \\
&\quad - \sigma(\alpha_1\rho_{G1}V_{G1}^2 + (1-\alpha_1)\rho_L V_{L1}^2) \\
&= \frac{1}{2}(\rho_{m1}^+ + \rho_{m2}) g l_2 H \cos\theta \\
&\quad - \frac{(1-x)^2}{\rho_L}\sigma^2 G_1^2 (1-\sqrt{\sigma})(l_2 - l_0)\lambda_{L2}(Re_L)\phi_{L2}^2(\chi^2)
\end{aligned}$$

$$\begin{aligned}
&(\alpha_2\rho_{G2}V_{G2}h_{G2} + (1-\alpha_2)\rho_L V_{L2}h_{L2}) \\
&\quad - \sigma(\alpha_1\rho_{G1}V_{G1}h_{G1} + (1-\alpha_1)\rho_L V_{L1}h_{L1}) \\
&\qquad + \frac{1}{2}(\alpha_2\rho_{G2}V_{G2}^3 + (1-\alpha_2)\rho_L V_{L2}^3) \\
&\quad - \frac{\sigma}{2}(\alpha_1\rho_{G1}V_{G1}^3 + (1-\alpha_1)\rho_L V_{L1}^3) = \sigma G_1 l_2 g H \cos\theta
\end{aligned}$$

$$\begin{aligned}
&\lambda_{L2}(Re_L)\frac{(1-x)^2 G_1^2 \sigma^2}{2D_2\rho_L}\phi_{L2}^2(\chi^2) \\
&\quad = (1-\alpha_2)(\rho_L - \rho_{G2}) g \cos\theta \\
&\quad + \frac{3}{8R_{B2}} f(\alpha_2) C_{DB2}(Re_B)\rho_L(V_{G2} - V_{L2})^2 \qquad (26)
\end{aligned}$$

where θ is the angle between $\mathbf{1}_z$ and $\mathbf{g}$ (Fig. 1), and

$$Re_B = \frac{2\rho_L(V_G - V_L)R_B}{\mu_L}$$

is the bubble Reynolds number.

The establishing length of the flow, L_2, is the length from which the distribution of the wall shear stress becomes uniform. Measurements made by Suleman [5] and Aloui [7] performed in a bubbly flow have shown that $L_2 \cong 8D_2$; it seems that L_2 weakly depends on the void fraction and the area ratio (for $\alpha < 0.30$ and $0.1 < \sigma < 0.45$). Further, measurements on the distribution of τ_D downstream from the enlargement made by Suleman have shown that the reduced characteristic length $l_0 \cong 15$. This characteristic length seems to weakly depend on the void fraction and σ (for $\alpha < 0.30$ and $0.1 < \sigma < 0.45$).

The algebraic set of equations (26) is highly nonlinear: its solution requires an iterative numerical procedure. The knowledge of the real global void fraction upstream from the enlargement is necessary to resolve the problem. The multiplier factor of the singular pressure variation will be defined by

$$\phi_{Lo}^2 = \frac{\Delta p_{sing}}{\Delta p_{sing,Lo}} \qquad (27)$$

where the two-phase singular pressure variation will be evaluated from p_2, deduced from the solution of the set of equations (26) by

$$\Delta p_{\text{sing}} = (p_2 - p_1) + \frac{(1-x)^2}{\rho_L} G_1^2 \sigma^2 (1-\sqrt{\sigma})$$

$$\times l_2 \lambda_{L2}(Re_L)\phi_{L2}^2(\chi^2) - \rho_{m2} g l_2 H \cos\theta. \quad (28)$$

Equation (28) is deduced from the extrapolation of the longitudinal pressure profile, which is obtained experimentally. By expressing equation (28) for the liquid only, one obtains

$$\Delta p_{\text{sing,Lo}} = \sigma(1-\sigma)\frac{G_1^2}{\rho_L} + \sigma^2(1-\sqrt{\sigma}) l_2 \lambda_{L2}(Re_L)\frac{G_1^2}{\rho_L}. \quad (29)$$

The gravity term does not appear any more in the expression of $\Delta p_{\text{sing,Lo}}$ due to the assumed incompressibility of the liquid phase: the liquid density is uniform downstream from the enlargement. Note that the solution of equations (26) requires assumptions H5 and H6.

5. SIMULATION RESULTS

For the MEM and MFM, the multiplier factor Φ_{Lo}^2 is calculated from equations (16) and (17). For the bubbly flow model, Φ_{Lo}^2 is calculated by equations (27)–(29). The three types of model will be compared in terms of Φ_{Lo}^2, based on the same conditions of pressure, temperature, mass velocity and mass quality upstream from the enlargement. These conditions will be fixed by the experimental data. Velasco [2] has obtained measurements of the pressure distribution and void fraction of an air–water bubbly flow along a sudden vertical enlargement under the following conditions:

$\sigma = 0.312$,
$p_1 \cong 1.5 \times 10^5$ Pa and $T_1 \cong 293$ K,
x varies between 7.1×10^{-4} and 3.4×10^{-3}
$(0.1 \leqslant \alpha \leqslant 0.5)$,
$V_{SL,1}$ varies between 2.8 and 4.4 m s^{-1}.

Calculations show that, by varying the superficial velocity of the liquid in the interval considered by the author, practically no significant influence is observed on Φ_{Lo}^2 for the three models considered here. This seems logical since the parameter Φ_{Lo}^2 is defined as the ratio of two quantities. Each of these quantities is approximately proportional to the square of $V_{SL,1}$ for a relatively low mass velocity (the compressibility effect is then small).

The void fraction will be determined by the experimental results of Velasco [2]. These results are plotted in Fig. 2(a) in terms of the upstream global void fraction as a function of the volumetric quality α_h defined by $Q_G/(Q_G+Q_L)$, where Q_K is the volume flow rate of phase K. It can be observed that the experimental data approximately form a straight line approached by the following correlation: $\alpha_1 = 0.71\alpha_h$, which fits the experimental data very well [Fig. 2(a)]. The solution

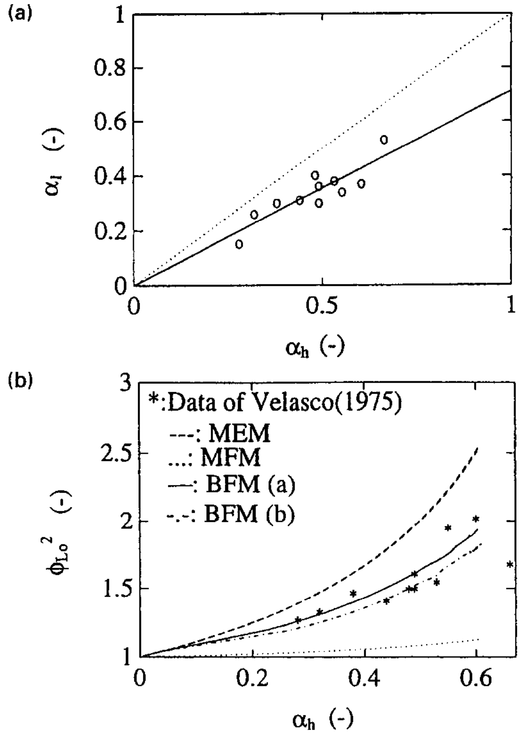

Fig. 2. (a) Upstream global void fraction as a function of the volumetric quality: (0) experimental data of Velasco [2], (· · ·) homogeneous conditions, (—) linear correlation $\alpha_1 = 0.71\alpha_h$. (b) Multiplier factor of the singular pressure variation as a function of the volumetric quality: (*) experimental data of Velasco [2], (- - -) MEM, (· · ·) MFM, (—) proposed BFM [$V_{SL,1} = 4.4$ m s^{-1} (a)], (-.-) proposed BFM [$V_{SL,1} = 2.8$ m s^{-1} (b)].

of the set of equations (26) also requires some closure laws for:

—the wall multiplier friction factor Φ_L^2: wall shear stress measurements of Souhar [10] have led to a correlation valid for a bubbly flow regime which agrees with his results with a better accuracy than the Lockhart–Martinelli correlation [11] (accuracy ~8%). Based on the measurements of the wall shear stresses, Suleman [5] proposes a correction to Souhar's correlation:

$$\phi_L^2 = 1 + \frac{40}{\chi} + \frac{400}{\chi^2}.$$

This expression will be used in the present calculations.

—the drag coefficient of a bubble C_{DB} (Ishii–Zuber's correlation [12] is used):

$$C_{DB} = \frac{24}{Re_B}\left(1 + \frac{Re_B^{3/4}}{10}\right)$$

for $N_{vi} < N_C$ (viscous regime)

$$C_{DB} = \frac{4}{3} R_B \sqrt{\frac{g(\rho_L - \rho_G)}{\xi}}$$

for $N_{vi} \geqslant N_C$ (deformed regime)

where

$$N_{vi} \equiv \frac{\mu_L}{\left\{\rho_L \xi \left[\frac{\xi}{g(\rho_L - \rho_G)}\right]^{1/2}\right\}^{1/2}}$$

is the viscosity number, and

$$N_C \equiv \frac{36\sqrt{2}}{Re_B^2}\left(1 + \frac{Re_B^{3/4}}{10}\right)$$

where ξ is the surface tension.

The single-phase friction factor is calculated by Blasius correlation. Solution of equation (26) is obtained by a Newton–Raphson iterative method. The three models compared with the experimental results of Velasco are shown in Fig. 2(b), where the volumetric quality has been taken as a variable. The two simplified models correspond to the limit of an area including all the experimental data. The MEM and MFM, respectively, overestimate and underestimate the singular multiplier factor deduced from the measurements. The overestimation of the MEM can be explained by the ideal momentum transfer between the phases. This assumption leads the liquid to decelerate when it passes through the expansion in the same way as a gas. Consequently, the decrease in the momentum of the mixture mainly contributed by the liquid (due to the small mechanical inertia of the gas, $\rho_G \ll \rho_L$) is more important than it is in reality. It leads to a pressure recovery which is also higher than in practice.

The underestimation of the MFM can be explained by the free movement of each phase, which corresponds mainly to its mechanical inertia. Due to its large inertia, the liquid will decelerate through the expansion much more easily than a gas. Consequently, the decrease in the momentum of the mixture is smaller than in reality. This induces a pressure recovery also smaller than in reality. When the bubble–liquid interaction is taken into account, the mixture passes the expansion with a change in momentum compressed between the values corresponding to the two simplified models. The calculations by the bubbly flow model (BFM) are performed for each limit of the superficial velocity of the liquid which characterizes the data ($V_{SL,1} = 2.8$ and 4.4 m s^{-1}). The BFM (26) presents a satisfying agreement with the measurements [Fig. 2(b)], and no significant velocity effect is predicted by the BFM. The bubble radius remains in principle a necessary parameter to apply model (26). The average bubble diameter generated in the Velasco tests is of the order of 2–3 mm, which means $R_{B1}/R_1 \cong 1/10$. This value has been introduced to solve the problem. By varying the reduced bubble radius from $\frac{1}{20}$ to $\frac{1}{5}$, the results of the bubbly flow model remain practically unchanged. Thus, for reduced bubble radii which remain compatible with the structure of a bubbly flow, model (26) seems to be rather insensitive to the bubble radius.

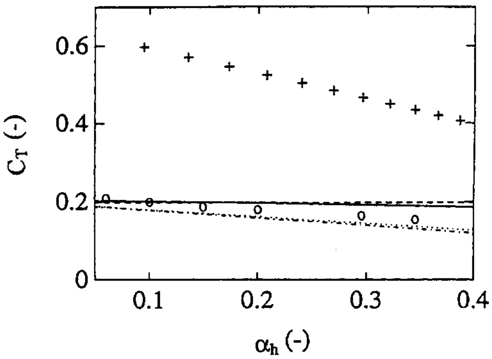

Fig. 3. Coefficient C_T as a function of the volumetric quality: (0) experimental data of Owen *et al.* [4], (---) MEM, (···) MFM, (-.-) model of Chisholm and Sutherland [1], (+ + +) model of Wadle [3], (—) proposed BFM.

Owen *et al.* [4] made some measurements of the pressure profile of a bubbly air–water flow along an horizontal sudden enlargement under the following conditions:

$\sigma = 1/9$,
$p_1 \cong 1.5 \times 10^5$ Pa and $T_1 \cong 293$ K,
$0 \leqslant \alpha \leqslant 0.35$.

The calculations have been made in the same way as above. For this particular set of experiments, the two phases were well mixed at the inlet of the enlargement, and, consequently, it is assumed that $\alpha \sim \alpha_h$. The results of the three models are close to each other. In Fig. 3, the experimental data of Owen *et al.* are plotted in terms of the coefficient C_T defined by $\Delta p_{sing}/(G_1^2/2\rho_H)$ as a function of α_h. The calculation of this parameter is insensitive to the values of the superficial velocity of the liquid. Thus, it is fixed in the simulations that $V_{SL,1} = 5$ m s^{-1}. Also plotted are the predictions of the following models:

$$C_t = 2\sigma(1-\sigma)\frac{\rho_H}{\rho_L}(1-x)^2\left(1 + \frac{C}{X_t} + \frac{1}{X_t^2}\right)$$

(Chisholm and Sutherland [1])

with

$$X_t = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{1/2}$$

and

$$C = \left[1 + \frac{1}{2}\left(\frac{\rho_L - \rho_G}{\rho_L}\right)^{1/2}\right]\left[\left(\frac{\rho_L}{\rho_G}\right)^{1/2} + \left(\frac{\rho_G}{\rho_L}\right)^{1/2}\right]$$

$$C_t = K\rho_H(1-\sigma^2)\left[\frac{x^2}{\rho_G} + \frac{(1-x)^2}{\rho_L}\right] \quad \text{(Wadle [3])}$$

with $K = \frac{2}{3}$ denoting a constant determined experimentally by Wadle.

One can see that the model of Chisholm *et al.* and the MFM underestimate the experimental coefficient C_T, although their predictions remain reasonably good. The predictions of these two models are close to each other. The underestimation of the Chisholm model has also been observed by other authors [2, 3], who have compared this model with other experimental data. Wadle's model widely overestimates the measured C_T coefficient. Owen *et al.* have observed that, with $K = 0.22$ (very different from $\frac{2}{3}$), Wadle's model agrees quite well with the measurements. The parameter K, unless it is submitted to an adequate modeling, makes Wadle's equation empirical. Regarding the BFM (26), it is shown that the agreement is satisfactory with the measurements of Owen *et al.*; also noted are some small overestimations of C_T for relatively high void fractions ($\alpha_h \geqslant 0.2$). The present model is not affected by any empirical correction. The predictions of the BFM are close to the results of the MEM due to the small slip velocity in the horizontal flow.

Suleman [5] has made some local measurements of the pressure profile and the void fraction of a bubbly air–water flow along vertical sudden enlargements under the following conditions:

$\sigma = 0.111$, 0.25 and 0.444,
pressure and temperature close to atmospheric conditions,
$0 \leqslant \alpha \leqslant 0.3$,
$1 \text{ m s}^{-1} \leqslant V_{SL,1} \leqslant 4 \text{ m s}^{-1}$.

These data are characterized by small mass velocities.

These experiments are simulated by the BFM. The results of the simulations have been compared with the data in Fig. 4. It is observed that the proposed model (26) fits these data very well.

The mean discrepancy between the predictions and the data are equal to 3.5% for $\Delta p_{sing} > 1800$ Pa. The error is greater for small values of ΔP_{sing} ($\sim 30\%$) corresponding to very small mass velocities. This can be explained by the greater uncertainty in the measurement of this quantity under these conditions; the fluctuations in the variables of the flow become significant.

There are only a few experimental data for two-phase bubbly flows through horizontal pipe enlargement. Local measurements have been made of pressure and void fraction profiles of a bubbly flow along an horizontal sudden enlargement under the following conditions:

$\sigma = 0.358$ ($D_1 = 0.017$ m and $D_2 = 0.0284$ m),
pressure and temperature close to atmospheric conditions,
$0 \leqslant \alpha \leqslant 0.3$,
$2.9 \text{ m s}^{-1} \leqslant V_{SL,1} \leqslant 5.6 \text{ m s}^{-1}$.

These data are characterized by $\Delta P_{sing} > 1700$ Pa.

The mass velocity of the liquid must be sufficiently high to avoid the stratification phenomena shown by several pattern maps. In the region of superficial liquid velocities investigated, the gas phase remains dispersed, and no stratification effect was observed downstream from the singularity. The results of the BFM calculations are compared with the present data in Fig. 5. One can observe that the bubbly model (26) is in good agreement with the present data. The large majority of the experimental points are predicted by the model with an error less or equal to 4%. These results (Figs. 4 and 5) are not sensitive for values of reduced bubble radius from $\frac{1}{20}$ to $\frac{1}{5}$.

6. CONCLUSIONS

The two-phase gas–liquid flow through a sudden enlargement is analysed by using the global balance

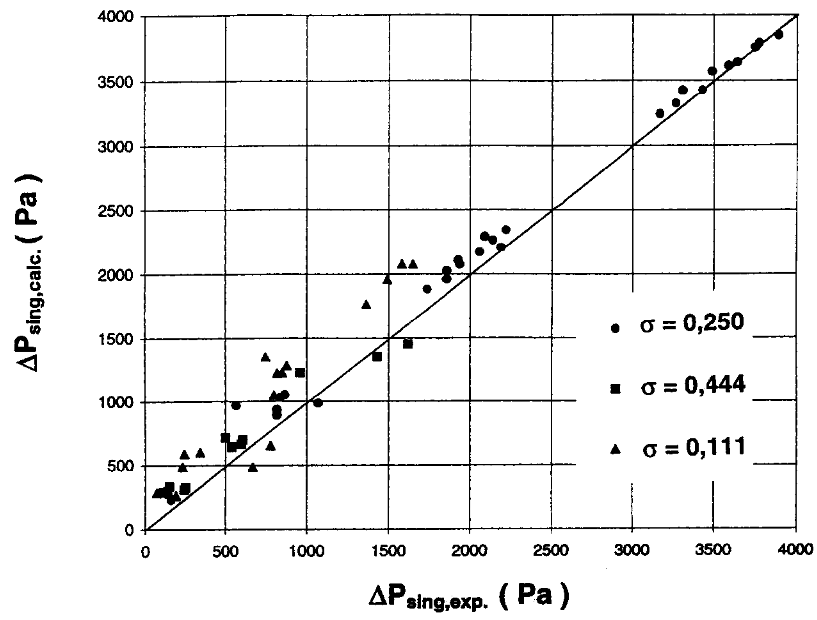

Fig. 4. Comparison of the proposed BFM with the experimental data of Suleman [5].

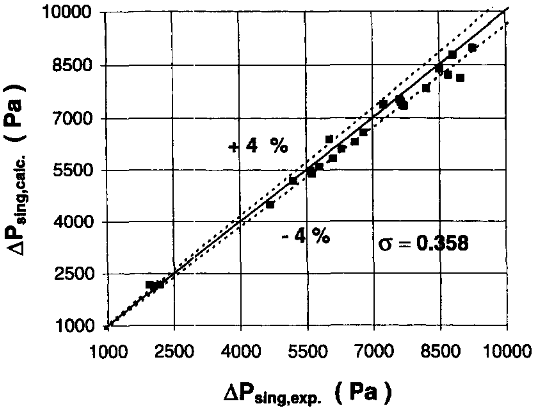

Fig. 5. Comparison of the proposed BFM with the new experimental data.

laws. The thermal equilibrium between the phases has been systematically accepted. The influence of the momentum transfer on the global characteristics of the expansion is studied by developing two limit models: the MEM and the MFM. The comparison between these two simplified models in terms of reduced singular pressure variation with some experimental data shows that these models simulate extreme conditions. A BFM which takes into account the drag of the bubble is developed, and fits the experimental data of the literature well. New experimental measurements of adiabatic air–water bubbly flows through an horizontal pipe enlargement have been performed. The proposed BFM is found to be in good agreement with the present data. It is found that this physical model is only weakly influenced by the reduced bubble diameter. This study shows the importance of the mechanical interaction between the phases when a two-component gas–liquid flow through an abrupt enlargement has to be modeled.

REFERENCES

1. Chisholm, D. and Sutherland, L. A., Prediction of pressure gradients in pipeline systems during two-phase flow. *Proceedings of the Institution of Mechanical Engineers*, 1969, **184**, Pt 3C, 24–32.
2. Velasco, I., L'écoulement diphasique à travers un élargissement brusque. Travail de maîtrise, Université catholique de Louvain, 1975.
3. Wadle, M., A new formula for the pressure recovery in an abrupt diffusor. *International Journal of Multiphase Flow*, 1989, **15**, 241–256.
4. Owen, I., Abdou-Ghani, A. and Amini, A. M., Diffusing a homogenized two-phase flow. *International Journal of Multiphase Flow*, 1992, **18**, 531–540.
5. Suleman, S. O., Contribution à l'étude d'un écoulement gaz–liquide dans un élargissement brusque. Thèse de doctorat, INPL, Nancy, 1990.
6. Delhaye, J. M., Basic equations for two-phase flow modelling. In *Two-phase Flow and Heat Transfer in the Power and Process Industries*, eds. A. E. Bergles, J. G. Collier, J. M. Delhaye, G. F. Hewitt and F. Mayinger. Hemisphere, New York, 1981, pp. 40–97.
7. Aloui, F., Etude des écoulements monophasiques et diphasiques dans les élargissements brusques axisymétrique et bidimensionnel. Thèse de doctorat, INPL, Nancy, 1994.
8. Berne, P., Contribution à la modélisation du taux de production de vapeur par autovaporisation dans les écoulements diphasiques en conduite. Thèse de doctorat, Ecole Centrale des Arts et Manufactures, 1983.
9. Wallis, G. B., *One-dimensional Two-phase Flow*. McGraw-Hill, New York, 1969.
10. Souhar, M., Etude du frottement pariétal dans les écoulements diphasiques en conduites verticales: cas des régimes à bulles at à poches. Thèse de doctorat, INPL, Nancy, 1979.
11. Lockhart, R. W. and Martinelli, R. C., Proposed correlation of data for isothermal two-phase two-component flow in pipes. *Chemical Engineering Progress*, 1949, **45**, 39–48.
12. Ishii, M. and Zuber, N., Drag coefficient and relative velocity in bubbly, droplet or particulate flows. *A.I.Ch.E. Journal*, 1975, **25**, 843.